

Inverse

Exercise 2.13

a. $A = \begin{pmatrix} -1 & i \\ 1+i & 0 \end{pmatrix}$

b. $B = \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

c. $C = \begin{pmatrix} 1 & -1 & i \\ 1+i & 0 & 0 \end{pmatrix}$

Solution Exercise 2.13

a. $A = \begin{pmatrix} -1 & i \\ 1+i & 0 \end{pmatrix}$

$$\left(\begin{array}{cc|cc} -1 & i & 1 & 0 \\ 1+i & 0 & 0 & 1 \end{array} \right) \xrightarrow{\frac{1}{2}-\frac{1}{2}iR_2 \leftrightarrow R_1} \left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{2}-\frac{1}{2}i \\ -1 & i & 1 & 0 \end{array} \right) \xrightarrow{1R_1 \rightarrow R_2} \left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{2}-\frac{1}{2}i \\ 0 & i & 1 & \frac{1}{2}-\frac{1}{2}i \end{array} \right) \xrightarrow{-1iR_2} \left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{2}-\frac{1}{2}i \\ 0 & 1 & -1 & i \end{array} \right)$$

$$\text{inv } A = \begin{pmatrix} 0 & \frac{1}{2}-\frac{1}{2}i \\ -1 & i \end{pmatrix}$$

b.

$$B = \begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$$

$$\left(\begin{array}{cc|cc} 1 & i & 1 & 0 \\ i & 1 & 0 & 1 \end{array} \right) \xrightarrow{-1iR_2 \leftrightarrow R_1} \left(\begin{array}{cc|cc} 1 & -1 & i & 0 \\ 1 & i & 1 & 0 \end{array} \right) \xrightarrow{R_1 \rightarrow R_2} \left(\begin{array}{cc|cc} 1 & -1 & i & 0 \\ 0 & 2i & 1 & i \end{array} \right) \xrightarrow{-\frac{1}{2}iR_2} \left(\begin{array}{cc|cc} 1 & -1 & i & 0 \\ 0 & 1 & -\frac{1}{2}i & \frac{1}{2} \end{array} \right) \xrightarrow{iR_2 \rightarrow R_1} \left(\begin{array}{cc|cc} 1 & 0 & \frac{1}{2} & -\frac{1}{2}i \\ 0 & 1 & -\frac{1}{2}i & \frac{1}{2} \end{array} \right)$$

$$\text{inv } B = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}i \\ -\frac{1}{2}i & \frac{1}{2} \end{pmatrix}$$

c.

$$C = \begin{pmatrix} 1 & -1 & i \\ 1+i & 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|cc} 1 & -1 & i & 1 & 0 \\ 1+i & 0 & 0 & 1 & 1 \end{array} \right) \xrightarrow{\frac{1}{2}-\frac{1}{2}iR_2 \leftrightarrow R_1} \left(\begin{array}{ccc|cc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2}i \\ 1 & -1 & i & 1 & 0 \end{array} \right) \xrightarrow{-1 R_1 \rightarrow R_2} \left(\begin{array}{ccc|cc} 1 & 0 & 0 & \frac{1}{2} & -\frac{1}{2}i \\ 0 & -1 & i & 1 & -\frac{1}{2} + \frac{1}{2}i \end{array} \right) \xrightarrow{iR_2}$$

$$\left(\begin{array}{cc|cc} 1 & 0 & 0 & \frac{1}{2} - \frac{1}{2}i \\ 0 & 1 & i & -\frac{1}{2} - \frac{1}{2}i \end{array} \right)$$

$$\text{inv } C = \begin{pmatrix} 0 & \frac{1}{2} - \frac{1}{2}i \\ i & -\frac{1}{2} - \frac{1}{2}i \end{pmatrix}$$

Exercise 2.14

a. $A = \begin{pmatrix} -1 & -1 & -3 \\ 1 & 2 & 2 \\ 0 & 5 & -4 \end{pmatrix}$

b. $B = \begin{pmatrix} 1 & 3 & 3 \\ -1 & -4 & -4 \\ -1 & 3 & 2 \end{pmatrix}$

c. $C = \begin{pmatrix} -1 & -3 & -4 \\ 1 & 4 & -3 \\ 1 & 4 & -4 \end{pmatrix}$

d. $D = \begin{pmatrix} 0 & 1 & -4 \\ -1 & -1 & 1 \\ -2 & -1 & -1 \end{pmatrix}$

e. $E = \begin{pmatrix} 1 & 1 & 1 \\ -3 & -4 & -1 \\ 2 & 4 & -3 \end{pmatrix}$

f. $F = \begin{pmatrix} 1 & -2 & -2 \\ 3 & -5 & -5 \\ 1 & 0 & 1 \end{pmatrix}$

Solution Exercise 2.14

a.

$$A = \begin{pmatrix} -1 & -1 & -3 \\ 1 & 2 & 2 \\ 0 & 5 & -4 \end{pmatrix}$$

$$\begin{array}{c} \left(\begin{array}{ccc|ccc} -1 & -1 & -3 & 1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & 5 & -4 & 0 & 0 & 1 \end{array} \right) \xrightarrow{-1 R_1} \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & -1 & 0 & 0 \\ 1 & 2 & 2 & 0 & 1 & 0 \\ 0 & 5 & -4 & 0 & 0 & 1 \end{array} \right) \xrightarrow{-1 R_1 \rightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & -1 & 0 & 0 \\ 0 & 1 & -1 & 1 & 1 & 0 \\ 0 & 5 & -4 & 0 & 0 & 1 \end{array} \right) \xrightarrow{\frac{1}{5} R_3 \leftrightarrow R_2} \\ \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & -1 & 0 & 0 \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 1 & -1 & 1 & 1 & 0 \end{array} \right) \xrightarrow{-1 R_2 \rightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & 1 & 3 & -1 & 0 & 0 \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 0 & -\frac{1}{5} & 1 & 1 & -\frac{1}{5} \end{array} \right) \xrightarrow{-1 R_2 \rightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 3\frac{4}{5} & -1 & 0 & -\frac{1}{5} \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 0 & -\frac{1}{5} & 1 & 1 & -\frac{1}{5} \end{array} \right) \xrightarrow{-5 R_3} \\ \left(\begin{array}{ccc|ccc} 1 & 0 & 3\frac{4}{5} & -1 & 0 & -\frac{1}{5} \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 0 & 1 & -5 & -5 & 1 \end{array} \right) \xrightarrow{-3\frac{4}{5} R_3 \rightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 18 & 19 & -4 \\ 0 & 1 & -\frac{4}{5} & 0 & 0 & \frac{1}{5} \\ 0 & 0 & 1 & -5 & -5 & 1 \end{array} \right) \xrightarrow{\frac{4}{5} R_3 \rightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 18 & 19 & -4 \\ 0 & 1 & 0 & -4 & -4 & 1 \\ 0 & 0 & 1 & -5 & -5 & 1 \end{array} \right) \\ \text{inv } A = \begin{pmatrix} 18 & 19 & -4 \\ -4 & -4 & 1 \\ -5 & -5 & 1 \end{pmatrix} \end{array}$$

b.

$$B = \begin{pmatrix} 1 & 3 & 3 \\ -1 & -4 & -4 \\ -1 & 3 & 2 \end{pmatrix}$$

$$\begin{array}{c} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ -1 & -4 & -4 & 0 & 1 & 0 \\ -1 & 3 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{1R_1} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ -1 & -4 & -4 & 0 & 1 & 0 \\ -1 & 3 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{1R_1 \rightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 \\ -1 & 3 & 2 & 0 & 0 & 1 \end{array} \right) \xrightarrow{1R_1 \rightarrow R_3} \\ \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 & 1 & 0 \\ 0 & 6 & 5 & 1 & 0 & 1 \end{array} \right) \xrightarrow{\frac{1}{6} R_3 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & -1 & -1 & 1 & 1 & 0 \end{array} \right) \xrightarrow{1R_2 \rightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & 3 & 3 & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & -\frac{1}{6} & 1\frac{1}{6} & 1 & \frac{1}{6} \end{array} \right) \xrightarrow{-3 R_2 \rightarrow R_1} \\ \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & -\frac{1}{6} & 1\frac{1}{6} & 1 & \frac{1}{6} \end{array} \right) \xrightarrow{-6 R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 1 & -7 & -6 & -1 \end{array} \right) \xrightarrow{-\frac{1}{2} R_3 \rightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 3 & 0 \\ 0 & 1 & \frac{5}{6} & \frac{1}{6} & 0 & \frac{1}{6} \\ 0 & 0 & 1 & -7 & -6 & -1 \end{array} \right) \xrightarrow{-\frac{5}{6} R_3 \rightarrow R_2} \\ \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 4 & 3 & 0 \\ 0 & 1 & 0 & 6 & 5 & 1 \\ 0 & 0 & 1 & -7 & -6 & -1 \end{array} \right) \text{inv } B = \begin{pmatrix} 4 & 3 & 0 \\ 6 & 5 & 1 \\ -7 & -6 & -1 \end{pmatrix} \end{array}$$

c.

$$C = \begin{pmatrix} -1 & -3 & -4 \\ 1 & 4 & -3 \\ 1 & 4 & -4 \end{pmatrix}$$

$$\begin{array}{l} \left(\begin{array}{ccc|ccc} -1 & -3 & -4 & 1 & 0 & 0 \\ 1 & 4 & -3 & 0 & 1 & 0 \\ 1 & 4 & -4 & 0 & 0 & 1 \end{array} \right) \xrightarrow{-1 R_1} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & -1 & 0 & 0 \\ 1 & 4 & -3 & 0 & 1 & 0 \\ 1 & 4 & -4 & 0 & 0 & 1 \end{array} \right) \xrightarrow{-1 R_1 \xrightarrow{+} R_2} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 1 & 4 & -4 & 0 & 0 & 1 \end{array} \right) \xrightarrow{-1 R_1 \xrightarrow{+} R_3} \\ \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 0 & 1 & -8 & 1 & 0 & 1 \end{array} \right) \xrightarrow{1 R_2} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 0 & 1 & -8 & 1 & 0 & 1 \end{array} \right) \xrightarrow{-1 R_2 \xrightarrow{+} R_3} \left(\begin{array}{ccc|ccc} 1 & 3 & 4 & -1 & 0 & 0 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array} \right) \xrightarrow{-3 R_2 \xrightarrow{+} R_1} \\ \left(\begin{array}{ccc|ccc} 1 & 0 & 25 & -4 & -3 & 0 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 0 & 0 & -1 & 0 & -1 & 1 \end{array} \right) \xrightarrow{-1 R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 25 & -4 & -3 & 0 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right) \xrightarrow{-25 R_3 \xrightarrow{+} R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -28 & 25 \\ 0 & 1 & -7 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right) \xrightarrow{7 R_3 \xrightarrow{+} R_2} \\ \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -4 & -28 & 25 \\ 0 & 1 & 0 & 1 & 8 & -7 \\ 0 & 0 & 1 & 0 & 1 & -1 \end{array} \right) \\ \text{inv } C = \begin{pmatrix} -4 & -28 & 25 \\ 1 & 8 & -7 \\ 0 & 1 & -1 \end{pmatrix} \end{array}$$

d.

$$D = \begin{pmatrix} 0 & 1 & -4 \\ -1 & -1 & 1 \\ -2 & -1 & -1 \end{pmatrix}$$

$$\begin{array}{l} \left(\begin{array}{ccc|ccc} 0 & 1 & -4 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 & 1 & 0 \\ -2 & -1 & -1 & 0 & 0 & 1 \end{array} \right) \xrightarrow{-\frac{1}{2} R_3 \leftrightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ -1 & -1 & 1 & 0 & 1 & 0 \\ 0 & 1 & -4 & 1 & 0 & 0 \end{array} \right) \xrightarrow{1 R_1 \xrightarrow{+} R_2} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & -\frac{1}{2} & 1\frac{1}{2} & 0 & 1 & -\frac{1}{2} \\ 0 & 1 & -4 & 1 & 0 & 0 \end{array} \right) \xrightarrow{1 R_3 \leftrightarrow R_2} \\ \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & -4 & 1 & 0 & 0 \\ 0 & -\frac{1}{2} & 1\frac{1}{2} & 0 & 1 & -\frac{1}{2} \end{array} \right) \xrightarrow{\frac{1}{2} R_2 \xrightarrow{+} R_3} \left(\begin{array}{ccc|ccc} 1 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 1 & -4 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 1 & -\frac{1}{2} \end{array} \right) \xrightarrow{-\frac{1}{2} R_2 \xrightarrow{+} R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 2\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & -4 & 1 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & \frac{1}{2} & 1 & -\frac{1}{2} \end{array} \right) \xrightarrow{-2 R_3} \\ \left(\begin{array}{ccc|ccc} 1 & 0 & 2\frac{1}{2} & -\frac{1}{2} & 0 & -\frac{1}{2} \\ 0 & 1 & -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -2 & 1 \end{array} \right) \xrightarrow{-2\frac{1}{2} R_3 \xrightarrow{+} R_1} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 5 & -3 \\ 0 & 1 & -4 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & -2 & 1 \end{array} \right) \xrightarrow{4 R_3 \xrightarrow{+} R_2} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2 & 5 & -3 \\ 0 & 1 & 0 & -3 & -8 & 4 \\ 0 & 0 & 1 & -1 & -2 & 1 \end{array} \right) \\ \text{inv } D = \begin{pmatrix} 2 & 5 & -3 \\ -3 & -8 & 4 \\ -1 & -2 & 1 \end{pmatrix} \end{array}$$

e.

$$E = \begin{pmatrix} 1 & 1 & 1 \\ -3 & -4 & -1 \\ 2 & 4 & -3 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ -3 & -4 & -1 & 0 & 1 & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array} \right) \xrightarrow[-\frac{1}{3}R_2 \leftrightarrow R_1]{\sim} \left(\begin{array}{ccc|ccc} 1 & 1 & \frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array} \right) \xrightarrow[-1]{R_1 \xrightarrow{+} R_2} \left(\begin{array}{ccc|ccc} 1 & 1 & \frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & 0 & 2 \\ 2 & 4 & -3 & 0 & 0 & 1 \end{array} \right) \xrightarrow[-2]{R_1 \xrightarrow{+} R_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & 1\frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & 0 \\ 0 & 1\frac{1}{3} & -3\frac{2}{3} & 0 & \frac{2}{3} & 1 \end{array} \right) \xrightarrow{\frac{3}{4}R_3 \leftrightarrow R_2} \left(\begin{array}{ccc|ccc} 1 & 1\frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & -\frac{1}{3} & \frac{2}{3} & 1 & \frac{1}{3} & 0 \end{array} \right) \xrightarrow[\sim]{\frac{1}{3}R_2 \rightarrow R_3} \left(\begin{array}{ccc|ccc} 1 & 1\frac{1}{3} & \frac{1}{3} & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & 0 & -\frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{4} \end{array} \right) \xrightarrow[\sim]{-1\frac{1}{3}R_2 \rightarrow R_1}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 0 & -1 & -1 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & 0 & -\frac{1}{4} & 1 & \frac{1}{2} & \frac{1}{4} \end{array} \right) \xrightarrow[-4]{R_3} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 4 & 0 & -1 & -1 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & 0 & 1 & -4 & -2 & -1 \end{array} \right) \xrightarrow[-4]{R_3 \xrightarrow{+} R_1} \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 16 & 7 & 3 \\ 0 & 1 & -2\frac{3}{4} & 0 & \frac{1}{2} & \frac{3}{4} \\ 0 & 0 & 1 & -4 & -2 & -1 \end{array} \right) \xrightarrow[2\frac{3}{4}]{R_3 \xrightarrow{+} R_2}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 16 & 7 & 3 \\ 0 & 1 & 0 & -11 & -5 & -2 \\ 0 & 0 & 1 & -4 & -2 & -1 \end{array} \right)$$

$$\text{inv } E = \begin{pmatrix} 16 & 7 & 3 \\ -11 & -5 & -2 \\ -4 & -2 & -1 \end{pmatrix}$$

f.

$$F = \begin{pmatrix} 1 & -2 & -2 \\ 3 & -5 & -5 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & -2 & -2 & 1 & 0 & 0 \\ 3 & -5 & -5 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow[\sim]{\frac{1}{3}R_2 \leftrightarrow R_1} \left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 1 & -2 & -2 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow[-1]{R_1 \xrightarrow{+} R_2} \left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & -\frac{1}{3} & 1 & -\frac{1}{3} & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{array} \right) \xrightarrow[-1]{R_1 \xrightarrow{+} R_3}$$

$$\left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & -\frac{1}{3} & -\frac{1}{3} & 1 & -\frac{1}{3} & 0 \\ 0 & 1\frac{2}{3} & 2\frac{2}{3} & 0 & -\frac{1}{3} & 1 \end{array} \right) \xrightarrow[5]{R_3 \leftrightarrow R_2} \sim \left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & -\frac{1}{3} & -\frac{1}{3} & 1 & -\frac{1}{3} & 0 \end{array} \right) \xrightarrow[3]{R_2 \xrightarrow{+} R_3} \sim \left(\begin{array}{ccc|ccc} 1 & -1\frac{2}{3} & -1\frac{2}{3} & 0 & \frac{1}{3} & 0 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & 0 & \frac{1}{5} & 1 & -\frac{2}{5} & \frac{1}{5} \end{array} \right) \xrightarrow[2]{R_2 \xrightarrow{+} R_1} \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & 0 & \frac{1}{5} & 1 & -\frac{2}{5} & \frac{1}{5} \end{array} \right) \xrightarrow{5R_3} \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 5 & -2 & 1 \end{array} \right) \xrightarrow[-1]{} R_3 \xrightarrow{+} R_1 \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 2 & 0 \\ 0 & 1 & 1\frac{3}{5} & 0 & -\frac{1}{5} & \frac{3}{5} \\ 0 & 0 & 1 & 5 & -2 & 1 \end{array} \right) \xrightarrow[-1]{} \frac{3}{5} R_3 \xrightarrow{+} R_2$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -5 & 2 & 0 \\ 0 & 1 & 0 & -8 & 3 & -1 \\ 0 & 0 & 1 & 5 & -2 & 1 \end{array} \right)$$

$$\text{inv } F = \begin{pmatrix} -5 & 2 & 0 \\ -8 & 3 & -1 \\ 5 & -2 & 1 \end{pmatrix}$$